GLOW DISCHARGE MEASUREMENT OF GAS DENSITY IN SUPERSONIC RAREFIED FLOW

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A relationship is obtained between the gas density in supersonic rarefied flow and the intensity of the negative glow discharge emission. Some results are presented of measurements of the density in the free stream and near a model. It is shown that the density measurement based on negative glow discharge emission is effective in the static pressure range $5 \cdot 10^{-2}$–$5 \cdot 10^{-4}$ mm Hg.

Because of the simplicity of the equipment used, glow discharge emission is widely used for visualization of rarefied flows in low-density wind tunnels (see, for example [1–2]). However, because of the complexity of the processes leading to initiation of emission the relationship between emission intensity and gas density has not been established. Therefore this technique makes possible only a rough estimate of the density distribution.

Flow visualization using negative glow discharge emission is described in [3], where the supersonic nozzle was used as the cathode. It was found that emission is excited by electrodes with an energy corresponding to the cathode potential drop. Because of electron scattering upon collision with the molecules, the electron concentration and emission intensity decrease with increasing distance from the nozzle. In the following the results of [3] are used as a basis for obtaining relations which permit calculating the gas density distribution in the flow from the emission intensity distribution. As in [3], the working gas is assumed to be nitrogen or air, although the technique can be used for other gases.

1. In the case of nitrogen or air and for sufficiently low pressure the negative glow spectrum consists of the bands of the first negative system of nitrogen. The intensity of the other spectral systems in the visible region can be neglected. It is well known that the intensity $i(v', v'')$ of the vibrational band corresponding to radiational transition from the vibrational level v' of the upper electron state to the vibrational level v'' of the lower electron state is written in the form

$$i(v', v'') = g(v') a(v', v'') h\nu(v', v''). \tag{1}$$

Here $a(v', v'')$ is the transition probability, h is Planck's constant, and $\nu(v', v'')$ is the radiation frequency. The excitation conditions are accounted for in the factor $g(v')$, which in the case of electron excitation is expressed in the form

$$g(v') = \sum_{v_1''} N(v_1'') n(v', v_1'')^2 \int_E R_e^2(r_{v', v_1''}, E) f(E) dE. \tag{2}$$

Here $N(v_1'')$ is the molecular concentration at the vibrational level v_1'' of the lower electron state, from which population of the level v' of the upper electron state is accomplished; n is the electron concentration; (v', v'') is the overlap integral; $R_e(r_{v', v_1''}, E)$ is the electron transition moment; $r_{v'v''}$ is the r-centroid; $f(E)dE$ is the fraction of electrons with energy between E and E + dE.

For not very high temperature ($\lesssim 1000°K$) we can assume that all the molecules are initially at the zero vibrational level of the ground electron state. We also assume that for the first several vibrational levels with comparatively high electron energy the dependence of R_e on the internuclear distance is negligibly small [4, 5]. Then

$$g(v') = N(0) n(v', 0)^2 \int_E R_e^2(E) f(E) dE. \tag{3}$$

The emission intensity I of the negative system of nitrogen equals the sum of the intensities of the individual bands, i.e.,

$$I = \sum_{v', v''} i(v', v'') = A(v', v'') N(0) n \int_E R_e^2(E) f(E) dE$$

$$A(v', v'') = \sum_{v', v''} a(v', v'') h\nu(v', v'')(v', 0)^2. \tag{4}$$

Thus the emission intensity is proportional to the molecular concentration (gas density) and the electron concentration. However, determination of the molecular concentration from the absolute radiation intensity involves several difficulties. Therefore we shall examine another possibility.

Let the z-axis be directed along the nozzle wall. For sufficiently low pressure we can neglect the electron energy variation along the z-axis within the region of observation [3, 6]. Then the relative change of the emission intensity as we move from the point z to the point z + dz is

$$\frac{dI(z)}{I(z)} = \left[\frac{1}{N(z)} \frac{dN(z)}{dz} + \frac{1}{n(z)} \frac{dn(z)}{dz}\right] dz. \tag{5}$$

Or, introducing in place of the molecular concentration N(z) the gas density $\rho(z)$, we obtain

$$\frac{dI(z)}{I(z)} = \left[\frac{1}{\rho(z)} \frac{d\rho(z)}{dz} + \frac{1}{n(z)} \frac{dn(z)}{dz}\right] dz. \tag{6}$$

The electron concentration in the beam decreases with distance from the nozzle, and the electron concentration gradient along the z-axis is defined as

$$\frac{dn(z)}{dz} = -\mu\rho(z) n(z), \tag{7}$$

where μ is the electron beam attenuation factor [3]. Considering (7), we obtain

$$\frac{d\rho(z)}{dz} - \frac{d \ln I(z)}{dz} \rho(z) = \mu\rho^2(z) \tag{8}$$

$$\rho(z) = \rho(z_0) \frac{I(z)}{I(z_0)} \left[1 - \mu\rho(z_0) \int_{z_0}^{z} \frac{I(z)}{I(z_0)} dz\right]^{-1}. \tag{9}$$

It is convenient to select the initial point z_0 on the following basis. Usually, in supersonic free flow we can find a sufficiently small segment Δz over which the density change is negligibly small. Then for this segment $d\rho/dz \approx 0$ and (8) can be written in the form

$$\frac{d \ln I(z)}{dz} = -\mu\rho. \tag{10}$$

Since in this case ρ is independent of z, the curve $\ln I(z) = f(z)$ is a straight line whose slope defines the value of the density ρ. To find the magnitude of the density at each point z with the aid of (9) it is advisable to take values of the quantities z_0, $I(z_0)$, and $\rho(z_0)$ corresponding to the final segment of the linear relation (10), i.e., at the point where the density gradient becomes nonzero and the curve $\ln I(z) = f(z)$ is no longer a straight line.

To determine the density at any point of the flow, we must substitute into the resulting formulas the values of the emission intensity at this point and at the preceding (along the z-axis) points. For planar flow it is possible to use the integral values of the emission intensity along the lines of observation. For the more complex three-dimensional distribution of the gas density and emission intensity the calculation of the density is more difficult. In particular, in the case of axial symmetry of the density distribution in supersonic flow and near a model the well-known technique for calculating axisymmetric nonhomogeneities can be used to find the density field.

In the case of photographic recording of the emission and when operating on the rectilinear part of the characteristic curve, the emission intensity can be expressed in terms of the photographic material density D(z). Then

$$I(z) / I(z_0) = 10^{[D(z) - D(z_0)]/\gamma}, \tag{11}$$

where γ is the photographic material contrast coefficient. Thus, in regions where the density depends on z

$$\rho(z)=\rho(z_0)\,10^{[D(z)-D(z_0)]/\gamma}\left[1-\mu\rho(z_0)\int\limits_{z_0}^{z}10^{[D(z)-D(z_0)]/\gamma}dz\right]^{-1}. \qquad (12)$$

For a region with constant density we obtain

$$\rho=-\frac{2.3}{\mu\gamma}\frac{dD(z)}{dz}=-\frac{2.3}{\mu\gamma}\frac{D(z_2)-D(z_1)}{z_2-z_1}. \qquad (13)$$

The points z_2 and z_1 are located within the region where $d\rho/dz = 0$ and the dependence of D on z is linear.

The value of the coefficient μ can be obtained with the aid of preliminary calibration experiments with known gas density. In studying flow past a model the density is usually determined in the form of the relative value, for example in the form ρ_∞/ρ (ρ is the freestream density). In this case there is no need for a preliminary measurement of the value of the coefficient μ; the value of $\mu\rho_\infty$ is calculated from the slope of the straight portion of the curve of D versus z and substituted into (12).

2. The density measurements were made in a low-pressure wind tunnel. The experimental setup was described in [3]. The working gas was air. Calibration measurements were first made to determine the coefficient μ. The value of μ depends on the cathode potential fall, which cannot be determined in each specific experiment. Therefore it is better to obtain the dependence of u_- on the voltage fall on a discharge gap. Figure 1 shows such an experimentally determined relation. The measurement error was about 5%. We see that the value of μ decreases with increase of the voltage. This phenomenon is in agreement with the generally accepted ideas on the dependence of the effective collision cross section of the electron with an atom (molecule) on its energy.

The measurement of gas density in the supersonic flow behind the shock at the model was made under conditions of not too low a pressure, when comparison of the measurement results with the values of the parameters calculated with the aid of the continuum relations is still possible. The model used was a circular cylinder placed crosswise in the flow. The cylinder was sufficiently long so that end effects did not disturb the flow parameters within the length of the emission zone along the viewing axis and the flow could be considered two-dimensional. The supersonic flow conditions were as follows: Mach number $M_\infty = 5$, static pressure $p_\infty = 5.5 \cdot 10^{-3}$ mm Hg, stagnation temperature was room temperature. If we take the cylinder radius as the characteristic dimension, then the Reynolds number $R_\infty = 280$ and the Knudsen number $K = 0.027$. Figure 2 shows a photograph of the flow past the model. The photographic measurements of the negative were made in the direction from the nozzle toward the model along the nozzle axis. Figure 3 shows a microphotogram of the emission (curve 1) and the density distribution (curve 2) along the stagnation line near the model.

The slope of the linear segment was used to determine the gas density in the supersonic stream. The density was $(5.2 \pm 0.8) \cdot 10^{-5}$ kg/m^3. The value of the static density in the stream, calculated on the basis of measurements with a total head probe, was $(6.0 \pm 2.1) \cdot 10^{-5}$ kg/m^3. The agreement can be considered fairly good. The gas density in the region of the stagnation point along the axis, referred to the freestream density, was calculated with the aid of (12) from the emission microphotogram shown in Fig. 3. The resulting curve, representing the compression shock profile, is also shown in Fig. 3. We note that the compression shock front is rather strongly diffused; this is characteristic for rarefied gas flows. Within the limits of the measurement error the gas density ratio reached behind the shock near the model corresponds to the value calculated using the Hugoniot-Rankine relation, which for $M_\infty = 5$ is 5. Thus the results of the experimental study are in agreement with the calculated values, which confirms the validity of the assumptions made and the suitability of the proposed technique for experimental determination of the rarefied flow spectrum near models.

The most complete picture of the gas density distribution in supersonic flow is obtained by plotting the density field. In so doing an important factor is whether the density field is calculated from a flow pattern obtained as a result of a single measurement or whether the study is made using the "point-by-point" technique. In the latter case we must maintain both the wind tunnel parameters and the experimental equipment parameters constant for a long time interval, which is not always possible. An important feature of the technique described is the possibility of photographing the flow pattern as a whole and then calculating the density at any point. The isodensity lines for the transverse flow past a cylinder obtained in this way for $M_\infty = 5$, $R_\infty = 57$, $K = 0.13$, are shown in Fig. 4, where ξ is the distance from the forward face

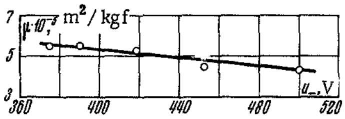

Fig. 1

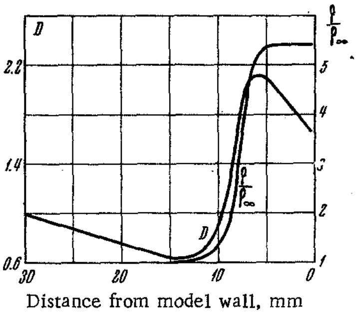

Fig. 3

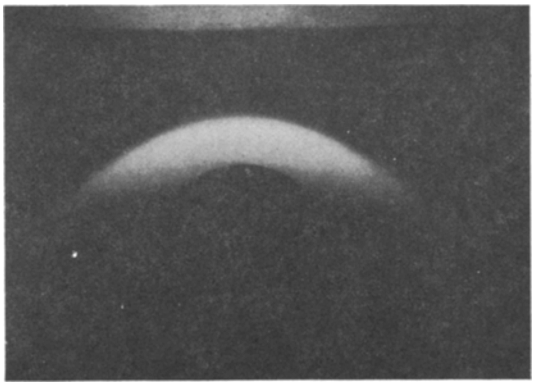

Fig. 2

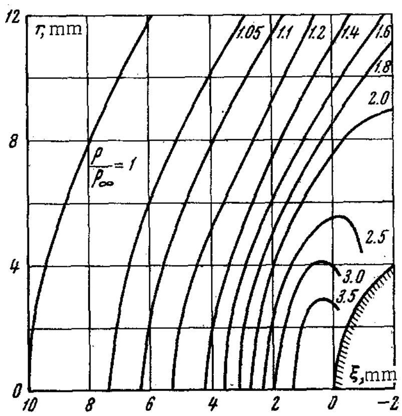

Fig. 4

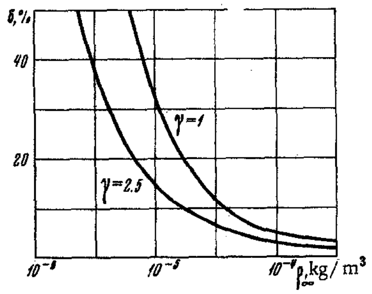

Fig. 5

of the cylinder along the stagnation line, r is the distance from the plane containing the axis of the cylinder and the stagnation line. Under certain conditions the density distribution can also be used to calculate the pattern of the other parameters.

3. The relative density measurement error

$$\delta = \frac{\Delta[\rho(z)/\rho_\infty]}{\rho(z)/\rho_\infty}$$

is determined primarily by the degree of a reflection and by the photographic material contrast coefficient γ (Fig. 5). The maximum possible measurement error when using a film with $\gamma = 2.5$ becomes large (about 25%) for $\rho = 5 \cdot 10^{-6}$ kg/m^3. This value, which corresponds to a static pressure of about $5 \cdot 10^{-4}$ mm Hg, can apparently be taken approximately as the lower limit in using the described technique. The upper limit is determined from the condition

$$\int_{z'}^{z''} \mu\rho\, dz \approx \mu \langle\rho\rangle (z'' - z') < 1$$

and depends on the experimental conditions. We note that in the case considered of density measurement near a cylinder the quantity $\mu\langle\rho\rangle(z''-z')$ is close to unity; however, the deviation of the measured value from the calculated value still does not exceed the measurement error.

An approximate limiting value of the pressure can be taken to be $5 \cdot 10^{-2}$ mm Hg. Thus, the technique for measuring the density in rarefied flows from the relative intensity of the negative glow discharge radiation can be used in the pressure range from $5 \cdot 10^{-2}$ to $5 \cdot 10^{-4}$ mm Hg.

REFERENCES

1. Physical Measurements in Gasdynamics and During Combustion [Russian translation], Izd-vo inostr. lit., Moscow, p. 82, 1957.

2. H. J. Bomelburg, "New glow discharge method for flow visualization in supersonic wind tunnels," J. Aero/space, vol. 25, no. 11, p. 727, 1958.

3. V. M. Kalugin, "Ultrasensitive glow discharge method for visualizing hypersonic flows of rarefied gases," PMTF [Journal of Applied Mechanics and Technical Physics], vol. 7, no. 4, 1966.

4. G. O. Langstroth, "The excitation of band systems by electron impact," Proc. Roy. Soc., vol. A146, p. 166, 1934.

5. D. R. Bates, "The intensity distribution in the nitrogen band systems emitted from the earth's upper atmosphere," Proc. Roy. Soc., vol. A196, no. 1044, p. 217, 1949.

6. A. K. Brewer and J. W. Westhaver, "The cathode region in the glow discharge," J. Appl. Phys., vol. 8, no. 11, p. 779, 1937.

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